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Theoretical developments for low energy experiments with radioactive beams *

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Abstract

In this talk I discuss two types of experiments with exotic nuclei which could be performed at the forthcoming Italian INFN facilities with radioactive beams. First I will discuss nuclear and Coulomb breakup experiments which involve heavy exotic beams and intermediate incident energies, thus being suited for the LNL-SPES proposed facility. Then I will discuss transfer to the continuum reactions aiming at performing spectroscopy in the continuum of light unbound nuclei like ^{10}Li. Such reactions are best matched if the incident beam energy is very low, as it will be at the LNS-EXCYT facility.

1 Introduction

Nuclei far from the stability valley are often called "exotic" because they exhibit properties rather different from those of nuclei in the rest of the nuclear chart [1]. Most of them are neutron rich and unstable against β-decay. It is interesting to study them because they give information on the structure of matter under extreme conditions and

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allow to test nuclear models otherwise based only on properties of stable nuclei.

Single particle degrees of freedom dominate both in the structure description as well as in the reaction studies of medium-light unstable nuclei. So far the most studied cases have been those of nuclei like ^{6}He, ^{11}Be ^{11}Li which exhibit the so called halo. ^{19}C is another interesting candidate still under investigation [2, 3, 4]. Heavier nuclei like ^{132}Sn have also attracted much attention. New techniques are needed to study these nuclei, which combine and unify the traditional treatment of bound and continuum scattering states [5]. Therefore, as in the early stages of Nuclear Physics, research on light exotic nuclei has concentrated on studying elastic scattering [6] and spectroscopic properties like the determination of single particle state energies, angular momenta and spectroscopic factors [2, 3].

2 Reaction models for structure studies: exclusive and inclusive breakup

One of the most suited measurement for an exotic projectile is the single-neutron removal cross section, in which only the projectile residue, namely the core with one less nucleon, is observed in the final state. This information together with the calculated cross sections [2, 3], has been used to extract single particle spectroscopic factors as in traditional transfer reactions. Besides the integrated removal cross section, denoted by σ_{-n}, the differential momentum distribution $d^3\sigma/dk^3$ is also measured. A particularly useful cross section is $d\sigma/dk_z$, the removal cross section differential in longitudinal momentum. It has been used to determine the angular momentum and spin of the neutron initial state [2] in a way similar to that proposed in [5, 8]. If the final state neutron can also be measured, the corresponding coincident cross section $A_p \rightarrow (A_p - 1) + n$ is called the diffractive (or elastic) breakup cross section if the interaction responsible for the removal is the neutron-target nuclear potential [9, 10]. In the case of heavy targets the coincident cross section contains also the contribution from Coulomb breakup due to the core-target Coulomb potential which acts as an effective force on the neutron. This observable is very useful to disentangle the reaction mechanism [11]. The difference between the removal and coincident cross sections is called the stripping (or absorption) cross section.

All theoretical methods used so far rely on a basic approximation to describe the collision with only the three-body variables of nucleon coordinate, projectile coordinate, and target coordinate. Thus the dynamics is controlled by the three potentials describing nucleon-core, nucleon-target, and core-target interactions. In most cases the projectile-target relative motion is treated semiclassically by using a trajectory of the center of the projectile relative to the center of the target $\mathbf{R}(t) = \mathbf{d} + \mathbf{v}t$ with constant velocity v in the z direction and impact parameter $\mathbf{d}$ in the xy plane.

A full description of the treatment of the scattering equation for a projectile which decays by single neutron breakup following its interaction with the target, including core recoil, can be found in [5, 11].

In ref.[11] it was shown that the combined effect of the nuclear and Coulomb interactions to all orders can be taken into account by using the potential $V = V_{nt} + V_{eff}$ sum of the neutron-target optical potential and the Coulomb dipole potential. If for the neutron final continuum wave function we take a distorted wave of the eikonal-type, then the amplitude for a transition from a nucleon bound state ϕ_i in the projectile to a final continuum state becomes :

$$A_{fi}\left(\mathbf{k},\mathbf{d}\right) = \frac{1}{i\hbar}\int d^3\mathbf{r}\int dt e^{-i\mathbf{k}\cdot\mathbf{r}+i\omega t} e^{\left(\frac{1}{i\hbar}\int_t^\infty V(\mathbf{r},t')dt'\right)} V\left(\mathbf{r},t\right)\phi_{l_i m_i}\left(\mathbf{r}\right) \tag{1}$$

where $\omega = \left(\varepsilon_f{}' - \varepsilon_0\right)/\hbar$ and ε_0 is the neutron initial bound state energy while $\varepsilon_f{}'$ is the neutron-core final continuum energy. Eq.(1) is appropriate to calculate the coincidence cross sections $A_p \to (A_p - 1) + n$ discussed in the previous section. Finally the differential probability with respect to the neutron energy and angles can be written as $\frac{d^3 P_{nc}(d)}{d\varepsilon_f{}' \sin\theta d\theta d\phi} = \frac{1}{8\pi^3}\frac{mk_n}{\hbar^2}\frac{1}{2l_i+1}\Sigma_{m_i}|A_{fi}|^2$, where A_{fi} is given by Eq.(1) and we have averaged over the neutron initial state.

The effects associated with the core-target interaction will be included by multiplying the above probability by $P_{ct}(d) = |S_{ct}(d)|^2$ the probability for the core to be left in its ground state, defined in terms of a core-target S-matrix function of d, the core-target distance of closest approach [10] .

Thus the double differential cross section is

$$\frac{d^2\sigma}{d\varepsilon_f{}' d\Omega} = C^2 S\int_0^\infty d\mathbf{d}\frac{d^2 P_{nc}(\mathbf{k},d)}{d\varepsilon_f{}'\Omega} P_{ct}(d), \tag{2}$$

and C^2S is the spectroscopic factor for the initial single particle orbital.

Inclusive cross sections in which only the core with $(A_p - 1)$ nucleons is detected need to take into account also the absorption of the neutron by the imaginary part of the n-target optical potential. For such reactions the Coulomb recoil effect can be neglected but the distorted eikonal-type wave function used in Eq.(1) is not accurate enough, in particular if the final continuum states are single particle resonances in the target plus one neutron nucleus. Then a distorted final neutron wave function, calculated by an optical model will be used. Also since the neutron is not detected one integrates over the neutron angles. Thus, according to [5] the final neutron probability energy spectrum with respect to the target reads

$$\frac{dP}{d\varepsilon_f} \approx \Sigma_{j_f}(|1-\bar{S}_{j_f}|^2+1-|\bar{S}_{j_f}|^2)(2j_f+1)(1+F_{j_f,j_i})\frac{C_i^2}{mv^2}\frac{1}{2k_f}\frac{e^{-2\eta d}}{2\eta d}M_{l_f l_i}, \tag{3}$$

where $\bar{S}_{j_f}$ is the neutron-target optical model S-matrix, F_{j_f,j_i} is an l to j coupling factor, η is the transverse component of the neutron momentum which is conserved in the neutron transition, d is the core-target impact parameter, C_i is the initial state asymptotic normalization constant and $M_{l_f l_i}$ is a factor depending on the angular parts of the initial and final wave functions, v is the relative motion velocity at the distance of closest approach.

3 Applications

We are going to discuss now a series of experiments and corresponding theoretical calculations aimed at extracting spectroscopic information on one-neutron and two-neutron halo nuclei and to determine properties of neutron-exotic nucleus interactions.

3.1 Neutron differential cross sections following breakup.

^{11}Be is probably the best known one-neutron halo nucleus since experimental information has been available for long time . The ground state is a $2s_{1/2}$ state with separation energy of 0.5MeV and spectroscopic factor $C^2S = 0.77$ [12]. Therefore it has been used as a test case for reaction models which use the above basic structure information as input [9, 10, 13].

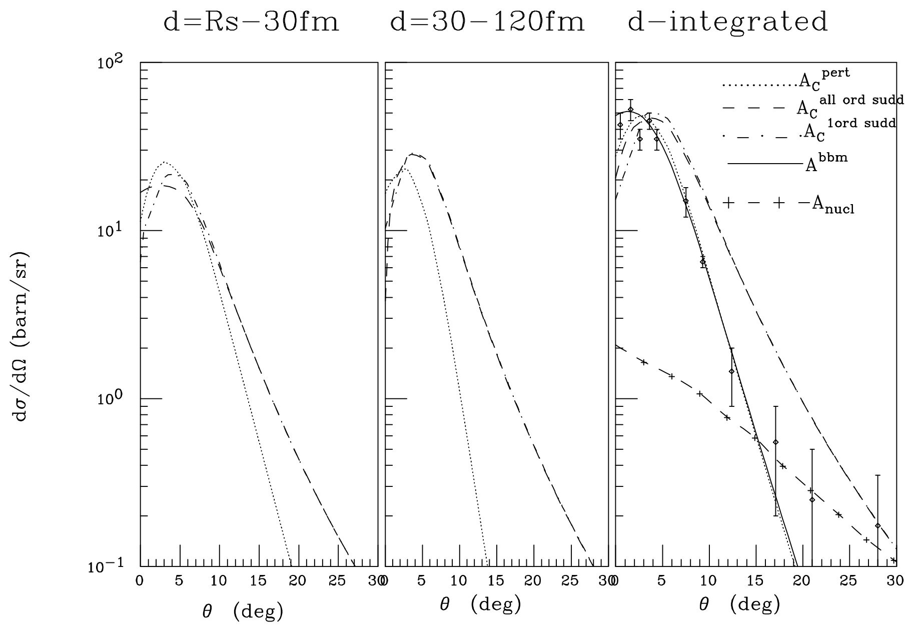

Figure 1: Neutron angular distributions distribution after nuclear-Coulomb breakup.

On the other hand a more recent work [11] has improved the previous knowledge of the breakup reaction, by studying the Coulomb-nuclear interference effects according to Eq.(1). We report here on new calculations with Eq.(1) to study higher order effects. Three limits of Eq.(1) have been tested. The first is the sudden approximation in which $\omega = 0$ and Eq.(1) can be calculated with nuclear and/or Coulomb to all orders. We call the corresponding amplitudes $A_C^{all\ ord\ sudd}$ and A_{nucl}. Then we have studied the first order approximation for the Coulomb term in which $e^{\left(\frac{1}{i\hbar}\int_t^\infty V_{eff}(\mathbf{r},t')dt'\right)} = 1$ but the ωt term is kept (this is the standard first order perturbation theory amplitude A_C^{pert}) and finally the sudden approximation restricted to first order giving $A_C^{1ord\ sudd}$. The main results of our new calculations are shown in Fig.(1a) and (1b) which give the neutron final angular distribution in the laboratory for the reaction ${}^{11}Be({}^{197}Au,{}^{197}Au){}^{10}Be{+}n$ at 41 A.MeV [9]. The curves in Fig.(1a) indicate that for $R_s < d < 30fm$

the results obtained with $A_C^{all\ ord\ sudd}$ are equal to those obtained with $A_C^{1ord\ sudd}$, starting from about $\theta = 10deg$ thus showing that higher order terms need to be considered only at small angles. On the other hand for $d > 30fm$ we find that higher order effects are always negligible since using $A_C^{all\ ord\ sudd}$ or $A_C^{1ord\ sudd}$ does not give any difference. We trust that higher order terms are calculated correctly by the sudden approximation because we have checked that the second order term calculated with full time dependence or in the sudden limit, gives the same results. Then we conclude that first order time dependent perturbation theory is valid and appropriate apart from the small impact parameter, small angle region. Thus in Fig.(1c) we finally give by the dotted curve the results of the simple first order perturbation theory while the solid curve is the all order calculation according to an amplitude defined as $A^{'bbm'} = A_C^{all\ ord\ sudd} + A_C^{pert} - A_C^{1ord\ sudd}$. Such an amplitude is valid at all core-target impact parameters, contains all order contributions and it does not give rise to any divergence in the final integral over impact parameters in Eq.(2) because the divergent term $A_C^{1ord\ sudd}$ is substituted by A_C^{pert}. Data points are from [9]. The theoretical calculations have been multiplied by the known spectroscopic factor. Analysis of the type presented in this section have been used and could be used in the future to extract spectroscopic factors.

Another neutron-rich nucleus which recently has been discussed at length [14] is ^{132}Sn which should exhibit the N=82 shell closure. It is expected that the spin-orbit splitting should decrease or even vanish for such very neutron-rich isotopes. A way to study the spectroscopy of this nucleus would be to measure the absolute cross sections and core parallel momentum distributions from the breakup of neutrons of different orbitals. Such a measurement would be possible by taking coincidences between the core and γ-rays as done in [2]. As one can see from Fig.(2) the shapes of the parallel momentum distributions, calculated according to Eqs.(2) and (3) depend on the initial state and are determined by the spin-orbit coupling coefficient F_{j_f,j_i} of Eq.(3). They reflect the momentum distribution of the neutron in the projectile and thus they and can be used to determine it. The ratio of the measured and calculated absolute cross sections on the other hand determine the single particle spectroscopic factor according to Eq.(2). We give the calculated values on top of each initial state spectrum in Fig.(2) together with the shell model occupation number.

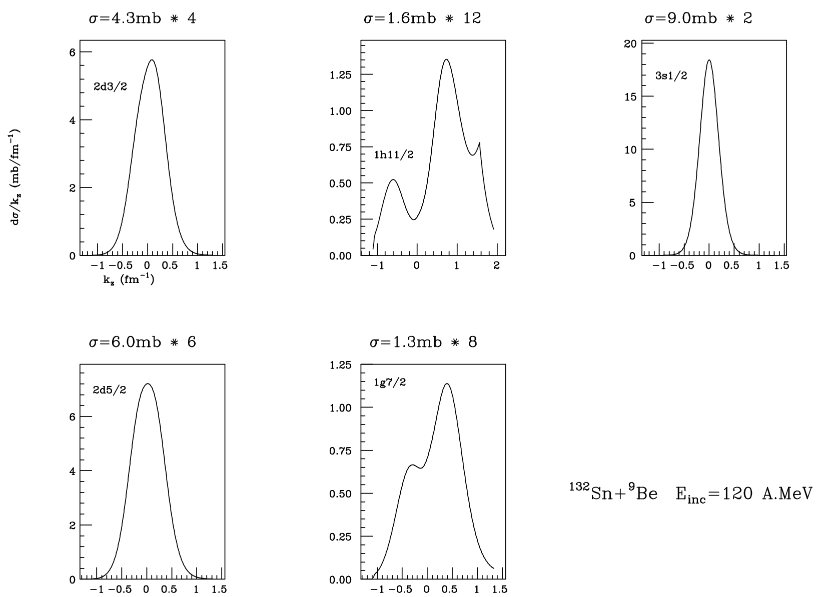

Figure 2: Neutron parallel momentum spectra following breakup of several single particle states in ^{132}Sn.

3.2 ^{10}Li spectrum and ^{11}Li properties.

We discuss now the results of a possible reaction aiming at clarifying the structure ^{11}Li which has been a challenge for long time [7, 15]-[18]. A similar reaction could be performed at the LNS with the ^{8}Li beam to study the structure of the unstable ^{9}Li and to determine the n-^{8}Li interaction.

^{11}Li and ^{6}He are two-neutron halo nuclei. Their corresponding A-1 systems, such as ^{10}Li are unbound and have therefore been very difficult to study from the experimental point of view [19]-[18]. However ^{11}Li is bound thanks to the pairing force acting between the two extra neutrons.

Recently the experiment $d(^{11}Be,^{3}He)^{10}Li$ [18] has confirmed that the ground state of ^{10}Li is a 2s virtual state.

Tree body models of ^{11}Li need as a fundamental ingredient the n-core (n-^{9}Li) interaction, which in turn determines the energies of the low energy unbound states in ^{10}Li. Following ref.[16] the two neutron hamiltonian is $H_{2n} = h_1 + h_2 + V_{nn}$. V_{nn} is the zero-range paring

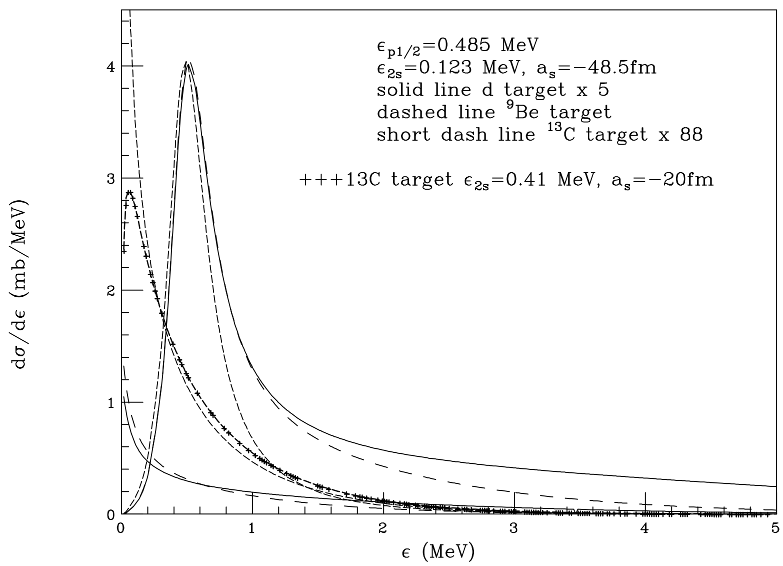

Figure 3: Neutron-^{9}Li relative energy spectra for transfer to the s and p continuum states in ^{10}Li.

interaction. The single neutron hamiltonian is $h = t + V_{cn}$ where t is the kinetic energy and $V_{cn} = V_{WS} + \delta V$ is the neutron-core interaction. It is given by the usual Woods-Saxon potential plus spin-orbit plus a correction δV which originates from particle-vibration couplings. They are important for low energy states but can be neglected at higher energies. If Bohr and Mottelson collective model is used for the transition amplitudes between zero and one phonon states, then $\delta V(r) = 16\alpha e^{2(r-R)/a}/(1 + e^{(r-R)/a})^4$ where $R \approx r_0 A^{1/3}$. According to [16] the best parameters for the n-^{9}Li Woods-Saxon and spin-orbit potential are $V_0 = -39.83MeV$, $V_{so} = 7.07MeV$, $r_0 = 1.27fm$, $a = 0.75fm$. The corresponding energies obtained for the 2s and 1p$_{1/2}$ states are given in Table 1, together with the values of the strength α of the correction potential δV.

It is therefore extremely important to determine experimentally the energies of the two unbound ^{10}Li states such that the interaction parameters can be deduced. Two $^9Li(d,p)^{10}Li$ experiments have re-

Table 1: Energies of the s and p states, width of the p-state, scattering length of the s-state and strength of the δV potential. (a) bound-state calculation, (b) scattering state calculation.

	(a)	(b)	Γ(MeV)	a_s(fm)	$\alpha(MeV)$
$\epsilon_{2s_{1/2}}(MeV)$	0.123	0.17		-48.5	-13.3
		0.45		-20	-14.0
$\epsilon_{1p_{1/2}}(MeV)$	0.485	0.595	0.48		3.3

cently been performed, at MSU at 20 A.MeV [20] and at the CERN REX-ISOLDE facility at 2 A.MeV[21]. For such transfer to the continuum reactions the predictions of the theory underlined in Sec. 2, Eq.(3) are very accurate. The sensitivity of the results on the target and on the energies assumed for the s and p states has been studied by calculating the reaction $^9Li(X, X-1)^{10}Li$ at 2 A.MeV for three targets d, ^{9}Be, ^{13}C. The ^{13}C target has been chosen because in such a case the neutron transfer to the 2s state in ^{9}Li would be a spin-flip transition which as it is well known are enhanced at low incident energy. For the other two cases the transfer to the 2s state is a non spin-flip transition which is hindered. Fig.(3) shows the neutron energy spectrum relative to ^{9}Li obtained with the interaction and single particle energies of Tables 1. In the case of the 2s virtual state also the scattering length $a_s = -\lim_{k\to 0} \frac{tan\delta_0}{k}$ is given. The peak of the p-state will determine without ambiguity the energy of the state in a target independent way. The width is modified by the reaction mechanism, but it can be deduced from the phase shift behavior once that the energy is fixed. There is a larger probability of population of the s-state in the spin-flip reaction initiated by the carbon target. A measure of the line-shape (or spectral function) and absolute value of the cross section will determine the energy of the state also in this case. The integral over energy of the distribution determines the spectroscopic factor of the state. There is no spreading width of the single particle state since the n-^{9}Li interaction is real at such low energies, the first excited state of ^{9}Li being at $E^* = 2.7MeV$. This means also that the "resonances" of the n-^{9}Li system are not compound nucleus resonances but rather elastic scattering resonances and therefore arise only from the term $|1-\bar{S}|^2$ in Eq.(3). The sensitivity of the model calculation on the energy of the state is shown in Fig.(3) by results indicated with the crosses obtained when for the the s-state $\varepsilon_{2s} = 0.45 MeV$ corresponding to $a_s = -20 fm$. A clear peak appears even if located at very small energy. The appearance of a peak in the transfer spectrum, depends on the behavior in Eq.(3) of the term $|1 - \bar{S}|^2$ which has always a maximum value equal to 4 at the energy of the state, with respect to the product of the other terms which have a divergent-like behavior as the energy approaches zero. We conclude that if a transfer to the continuum experiment could measure with sufficient energy resolution the line-shapes or energy distribution functions for the s and p-states in ^{10}Li our theory would be able to fix unambiguously the energies of the states. Those in turn could be used to test microscopic models of the n-^{9}Li interaction.

4 Conclusions and future challenges

Physics with radioactive beams is an extremely fascinating field in which the interplay between the understanding of the nuclear structure and that of the reaction mechanism is very strong and an enormous number of progress has been made in the last few years. However a number of improvements both experimental as well as theoretical need to be pursued. We have shown that there are experiments which could be performed at the forthcoming facilities at the Italian Nuclear Physics Laboratories for which the theoretician's community is ready to give the appropriate support.

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